CliqueR: A Graph Theory Game

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The game CliqueR is a pencil-and-paper game with a huge diversity of possible boards. This article defines the game and establishes many of its graph-theoretic properties. These properties point toward a natural technique for classifying the strategic depth of different starting boards. A system for automatic content generation of CliqueR boards is given and used to produce illustrations of the various properties of different boards. A number of variations on the game are proposed as well as possible improvements to the automatic design technique.

1 Introduction

THE game CliqueR was designed to familiarise students with certain concepts from graph theory. Once designed, the game showed a surprising strategic depth and also was an excellent domain for search based procedural content generation [1, 2]. This article will give the rules of CliqueR and the motives for its initial development, explain the graph theory that underlies the design of CliqueR boards, and sketch a system for automatically generating CliqueR boards with controlled properties.

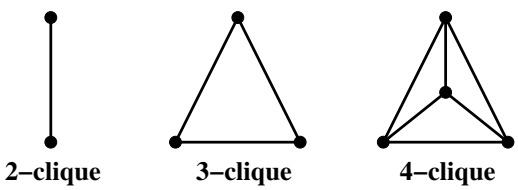

Figure 1. The three scoring configurations.

An example of a CliqueR board appears in Figure 2. A board consists of a placement of dots on a square with the property that no three of the dots lie on the same line. In order to explain the rules of CliqueR, we define the term clique. A *clique* is a set of dots such that every pair of dots is connected by an edge. Examples of two, three, and four vertex cliques are shown in Figure 1.

A mathematical fact that informs the design of CliqueR is that it is impossible to draw a 5-clique (five vertices with all possible connections) without at least one pair of edges crossing [3]. A move in CliqueR consists of drawing lines between dots, such that those lines do not cross other lines. Permitted moves for CliqueR are ones that eventually create the cliques shown in Figure 1. The rules are given below.

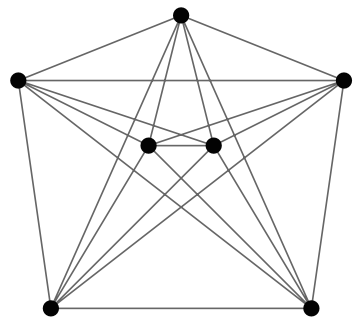

Figure 2. A CliqueR board with seven vertices.

CliqueR

1. Decide who moves first (e.g. by coin toss).

2. Players take turns moving if possible.

3. A move consists of drawing a line between two unconnected dots.

4. The line must not cross another line.

5. Completing a *clique* scores: 2-clique = 1 pt, 3-clique = 3 pts, 4-clique = 5 pts.

6. All cliques completed per move count.

Notice that drawing a line scores one point, since that is completing a 2-clique. Rule 6 means that if you complete a 3-clique, you get one point for the 2-clique (the line) and three more for completing the 3-clique. Similarly, completing a 4-clique requires you not only to draw the line that completes it but also to complete two 3-cliques as part of completing the four clique for a total of 1+3+3+5=12 points. Some sequences of play for the board shown in Figure 2 permit one move to complete two four-cliques for a total score of 20 points – this happened during play-testing.

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