The Complex 3×3×3

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A Rubik's Cube is a 3×3×3 twisty puzzle. This puzzle can be conceptualised as an array of 27 cubies. The 27 cubies consist of four types of pieces: eight corners, twelve edges, six faces and one core. But is this really the complete picture? Have we really accounted for all possible piece types? 3×3×3=27; thus, one is tempted to say 'yes', but this paper will argue that we are only seeing part of the picture. It will claim that the complete picture contains 64 cubies across ten piece types. These new cubies are defined, and a method for making them visible is detailed. This paper also explains why the resulting puzzle has been named the Complex 3×3×3.

1 Introduction

THE 3×3×3 twisty puzzle was invented by Ernő Rubik[1] in 1974 and marketed by Ideal Toys as the Rubik's Cube in 1980. This kicked off the exploration of twisty puzzles in general, and many different variations resulted. The 4×4×4 and 5×5×5 soon followed [1], as did the application of the same principles to many other geometries.

Previously we have seen how the 3×3×3 can be adapted to higher dimensions [2, 3] and shown how pieces normally hidden [4, 5] within the interior volume of the puzzle can be made visible [6]. Here the intent is to go back and take a closer look at the 3×3×3 itself.

The 3×3×3 is defined by six cut planes. If the 3×3×3 is centred on the origin, these planes are all an equal distance from the origin. It is valid to think of these planes as the $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 1$ and $z = -1$ planes. These planes cut all of 3-space up into 27 separate volumes. So it is apparent that if there is more to a 3×3×3, those pieces are not hiding inside or even outside the current surface of the puzzle. These 27 cubies constitute the set of *real*[2] pieces. But this is just the beginning of this investigation. This paper outlines several new piece types that represent new classes of cubies, their operation, and how they can be visualised.

2 Nortmann's Twistability

In 2009, Andreas Nortmann started the thread 'Analysis of Twistability and Virtual Pieces' on TwistyPuzzles.com, in which he introduced his scheme for analysing twisty puzzles [7]. To see how his approach can find additional pieces, we will apply it to the Skewb, shown in Figure 1.

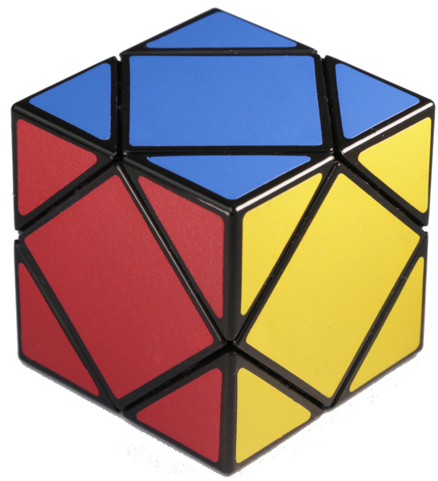

Figure 1. A Skewb.

The Skewb, first produced by Mefferts in 1982, is defined by four *cut planes*. Each plane perpendicularly bisects one of the four body diagonals of a cube, so all four planes cross at the origin. These four planes cut 3-space up into 14 volumes which map to the eight corners and six face pieces seen on the Skewb. These 14 pieces are all the real pieces possible in a Skewb. A corner rotation of a Skewb moves half of these 14 pieces (four corners and three faces). A single corner piece cannot be rotated by itself.

Andreas's methodology looks at the number of *linearly independent layers* per axis. It then defines a *holding point* for each possible set of linearly independent layers for a given puzzle. This is the point that is invariant, or stationary, under all rotations of the linearly independent layers. In some cases, it can be considered the intersection of all the dependent layers.

For the Skewb, let us name the six faces of the puzzle: R=*right*, L=*left*, U=*up*, D=*down*, F=*front*, and B=*back*. The RUF corner would then be the

[1]https://www.jaapsch.net/puzzles/patents/hu170062.pdf
[2]Key terms are defined in the Glossary.