How to Make a Better 3×3×3×3

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A Rubik's Cube is a 3×3×3 twisty puzzle. This puzzle can conceptually be pictured as an array of 27 cubies. However the standard Rubik's Cube only has 26 of these cubies stickered. Not all of the 26 stickered cubies have a fixed position and orientation in the solved state. Similar limitations are also true for the standard model of the 3×3×3×3. This paper discusses the methods used to overcome these limitations of the standard Rubik's Cube, the advantages and disadvantages of these methods, and how to apply one of the methods to the 3×3×3×3.

1 Introduction

In 1974, Ernő Rubik invented the 3×3×3 twisty puzzle. Ideal Toys produced and marketed his creation as the Rubik's Cube in 1980, and it took the world by storm. Not only were kids and adults fascinated by the puzzle, but it also caught the eye of mathematicians who saw the puzzle as an excellent representation of an algebraic group. The immense popularity of the Rubik's Cube sparked the creation of many variations: the 4×4×4, the 5×5×5, and the application of the basic principles to tetrahedral, dodecahedral, and other geometries as the population clamoured for more.

But mathematicians did not stop there; they saw the Rubik's Cube as a subject of mathematical investigation. Books were written that used the Rubik's Cube as an introduction to group theory [1]. Other mathematicians went on to consider Rubik's Cube variations which could not be placed on store shelves. The 3×3×3×3 was independently studied by Dan Velleman [2]; H. R. Kamack and T. R. Keane [3]; and Joe Buhler, Brad Jackson and Dave Sibley [4, 5]. While two of these papers are officially unpublished, they are currently available online and were directly available from the authors at the time they were written. So they are heavily referenced in other works. A prime example is the July 1982 column of 'Metamagical Themas' [6] in *Scientific American*, which was likely the first public exposure of the 3×3×3×3 in a mainstream publication.

The advent of 3D printing has led to a resurgence in the innovation of twisty puzzles [7]. New puzzles and new designs appear weekly and are discussed in online forums.[1] But many of these new innovations have not yet been carried over into the applications that allow one to play with the higher dimensional puzzles like the 3×3×3×3. For example, there are many methods used to turn a normal 3×3×3, or Rubik's Cube, into a Super Multi 3×3×3. In this context, *super* means that all stickered pieces of the puzzle have been given a fixed position and orientation in the solved state,[2] and *multi* means that all volumes created by the cut planes which define the puzzle are included as stickered pieces in the puzzle.[3] The normal 3×3×3 fails in both of these areas as the face centres are stickered but the four orientations that they can reach are indistinguishable and the 27^{th} or core cubie is not stickered at all. This paper will examine several of the common ways used to overcome these limitations and show how one of the most innovative methods can be extended to the 3×3×3×3.

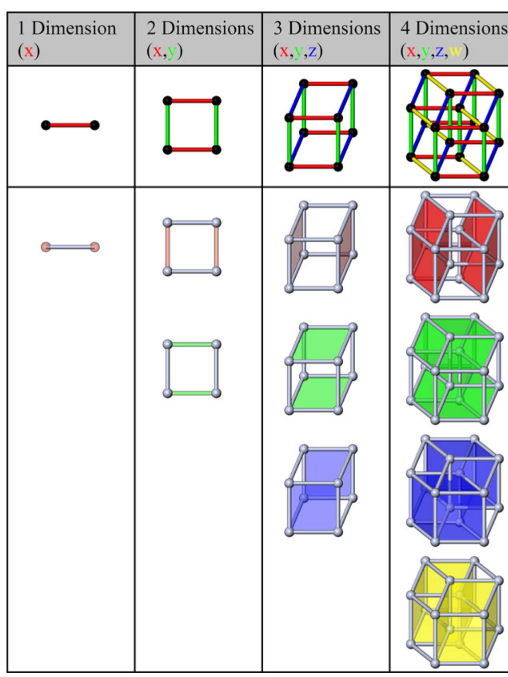

Figure 1. Line, square, cube and hypercube.

[1] http://www.twistypuzzles.com/
[2] http://twistypedia.oskarvandeventer.nl/index.php/Super-(puzzle_name)
[3] http://twistypedia.oskarvandeventer.nl/index.php/Multi-(puzzle_name)

Hoff, C., 'How to Make a Better 3×3×3×3', *Game & Puzzle Design*, vol. 2, no. 2, 2016, pp. 41–54. © 2016