From Computer Operations to Mechanical Puzzles

M. Oskar van Deventer, Freelance Mechanical Puzzle Designer

Igor Kriz, Department of Mathematics, University of Michigan

This article is about transforming a virtual puzzle implemented by computer operations into a physical puzzle. We describe why a puzzle motivated by a mathematical idea of the sporadic simple Mathieu group M_{12} is an interesting concept. We then describe how it was turned into two different physical puzzles called Topsy Turvy and Number Planet. Finally, we will show one version of a practical algorithmic solution to the puzzle, and describe challenges that remain.

1 Introduction

Ever since E. Rubik invented his famous cube in 1974, a boom of *twisty puzzles* began. The puzzles which emerged are too numerous to list here (for some information, see [1, 2]).[1] However, after an algorithm by David Singmaster for Rubik's cube was published in 1981 [3], it soon became clear that only small modifications of the strategy for finding the algorithm for Rubik's cube also produce algorithms for the other twisty puzzles.

The meta-strategy is as follows: by repeating some short sequences of moves, often commutators, i.e. moves of the form $aba^{-1}b^{-1}$ where a, b are simpler moves, one eventually finds moves which leave all but a small number of pieces in place. Additionally, positions of pieces affected by the move can be customised by finding setup moves, i.e. sequences of moves which get other pieces to the places where the original move took place, performing the move, and then backing out the setup sequence to move the chosen pieces instead. Invariably, with such generalised moves, which move only chosen small sets of pieces, the puzzle can be then solved in an incremental way.

Has the puzzling potential of twisty puzzles therefore been exhausted, i.e. is there no twisty puzzle which would require a completely different strategy? Algebra suggests that this should not be the case, and that substantially new kinds of puzzles should exist. The mathematics of a puzzle is determined by its *permutation group* [4], which is basically the set of all possible moves one can make on the puzzle. In all of the twisty puzzles on the market, the permutation group consists of almost all permutations, with restrictions on only a few types, which do not alter the strategy for solving the puzzle substantially. However, in mathematics, we know that some very strange permutation groups exist. The strangest ones are called *sporadic simple groups* [5]. The word 'simple' here refers to the fact that they are building blocks for finite groups, just as prime numbers are basic building blocks of natural numbers. However, it may be a misnomer to call those groups simple, since they are typically extremely complicated.

Could one make a twisty puzzle based on a sporadic simple group, which would then require a completely novel strategy, different in essential ways from Singmaster's algorithm? The problem is that sporadic simple groups usually contain very strange permutations which often arise from higher mathematics, but do not have any easy interpretation in the real world. The idea of making a hand held puzzle from one of these groups certainly seemed out of reach.

2 Computer Implementation

In 2008, the second author, along with University of Michigan student Paul Siegel, investigated the question of whether a mechanical model of a puzzle was really necessary to make it interesting. Could a puzzle be a computer program, where the player would, by moving or clicking a mouse, instead of twisting a physical puzzle, execute a move? Would anyone be interested in solving such a puzzle?

They soon realised that interesting computer puzzles could be made with several sporadic simple groups.[2] The simplest, and perhaps most popular, such puzzle used the Mathieu 12 (M_{12}) sporadic simple group. This puzzle has two basic moves, Invert and Merge, as indicated in Figure 1. The Invert move reverses the order of the numbers. The Merge move is akin to a card shuffle. The object of the M_{12} puzzle is similar to the object of the Rubik's Cube: scramble and solve by using only the two permutations.

[1] https://en.wikipedia.org/wiki/Combination_puzzle lists many such puzzles.
[2] http:www.math.lsa.umich.edu/~ikriz

van Deventer, M. O. and Kriz, I., 'From Computer Operations to Mechanical Puzzles', *Game & Puzzle Design*, vol. 2, no. 2, 2016, pp. 9–16.