Graph Theory in Game and Puzzle Design

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Almost any game or puzzle played on a board involves one or more abstract combinatorial graphs. This column introduces some basic terminology and intuition from graph theory, and describes some graph-based puzzles and games. The relationship between games and graphs is outlined with several examples, and the use of graph algorithms to aid in game design is discussed.

1 Introduction

GRAPH theory has historically been an orphan step-child of mathematics, a field that has received very little respect. A good deal of the progress in graph theory has been made by recreational, as opposed to professional, mathematicians, adding to the feeling it was not serious.

However, with the advent of the computer age, graph theory has proven most useful, with graphs serving as a language for designing and discussing computer networks, transportation networks, and even biological networks such as food webs and interaction patterns of biomolecules. Graph theory has now become a respected and practical branch of mathematics.

What is the relevance of graph theory to games and puzzles? Almost any game or puzzle played on a board implicitly involves at least one *abstract combinatorial graph*, which is a term that refers to the relationships between locations, objects, concepts and actions in the game. This article is intended to serve as an introduction to such graphs, which are often woven into the fabric of the game.[1] We will explore graph theory by scrutinising puzzles and games from 1736 to the present, and we will see how recognising and using this theory as a tool can aid in a more thorough and careful design process, as well as more balanced and intentional designs.

2 Reference Games

In this section, we introduce two historical examples of graph theory in a puzzle and in a game.

2.1 The Bridges of Königsberg

In the city of Königsberg, corresponding to modern day Kaliningrad, the citizens had an after-dinner activity. A river with a substantial island runs through the city and seven bridges joined the north shore, the island, the south shore, and the upstream angle between two branches of the river. The goal of the activity was to cross each bridge exactly once and return to start. This problem is shown graphically in Figure 1.

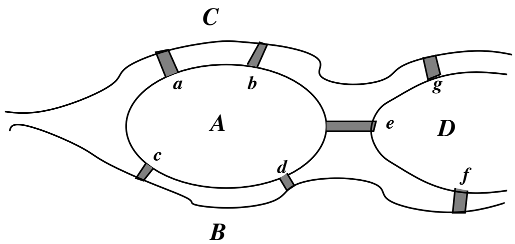

Figure 1. The Seven Bridges of Königsberg.

The great mathematician Euler proved that this problem has no solution [3]. His solution contained the seeds to a rapid technique for establishing the solvability of a whole class of puzzles and an effective technique for constructing solutions. This technique and the class of problems are described further in Section 4.

2.2 Icosian

Mathematician William Rowan Hamilton devised the game Icosian,[2] shown in Figure 2.

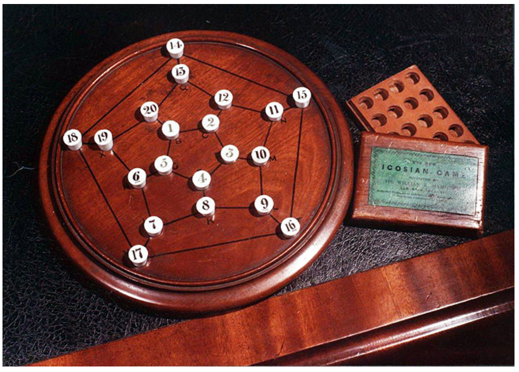

Figure 2. The original Icosian board (picture from the Library of the Royal Irish Academy).

[1]For more mathematically detailed coverage of this topic, see the classic *Winning Ways for your Mathematical Plays* [1] and Richard Guy's 'Graphs and Games' [2, chap. 9].

[2]https://www.ria.ie/library

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