Deriving Card Games from Mathematical Games

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In a mathematical game, such as the Prisoner's Dilemma or Rock-Paper-Scissors, players are permitted to choose from the available moves each turn. When that free choice of moves is replaced by cards with moves printed on them from a fixed deck, the mathematical game becomes a card game. This transformation often yields games of very different character, giving a simple technique for generating card-game mechanics from existing – and novel – mathematical games. We explore ways to translate games' payoff matrices into scoring systems, and an evolutionary algorithm to assess their resulting balance.

1 Introduction

MATHEMATICAL games are a staple of economics, planning, and academic game theory. Possibly the best known of these games is the *Prisoner's Dilemma* (PD) [1], invented by Melvin Dresher and Merrill Flood of the RAND Corporation, and placed in a prison setting by mathematician Albert Tucker in 1950. The game is used to model cooperation and conflict and, in its single-shot version, shows why rational agents might avoid cooperating.

The mechanics of the PD are simple. Two agents choose, simultaneously, to *cooperate* or *defect*. If both players cooperate, they receive the *cooperation* payoff **C**. If both players defect, they receive the *defection* payoff **D**. If the agents' moves are not the same, then the cooperator receives the *sucker* payoff **S** and the defector receives the *temptation* payoff **T**. In order to be considered PD, these payoffs must obey the following inequalities:

$$S < D < C < T \qquad (9)$$

and:

$$S + T < 2C \qquad (10)$$

If the game is iterated, i.e. repeated, and if the number of rounds is not known to the participants, then this game can be used to model how cooperation arises between rational agents. Its *payoff matrix* can be represented as a *directed graph*, whose vertices represent available moves and whose edges are weighted with the score that the move at the tail of the arrow yields, when played against the move at the head of the arrow.

Figure 1 shows this digraph for PD. States are labelled with moves and edges are annotated with scores for the tail of the arrow playing against the head. For two-move games, there is relatively little advantage to this technique, and this figure is included for illustrative purposes. The digraph becomes more useful when a game has more moves.

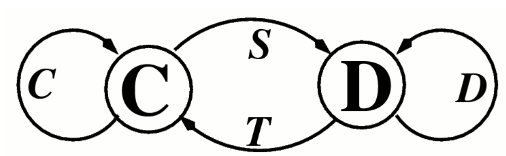

Figure 1. The digraph representation of the payoff matrix from PD.

In this article, we examine the impact of placing a restriction on which moves can be made in the form of hands of cards with moves printed on them. This mechanic permits mathematical games to be reincarnated as card games. In addition, transferring the moves to cards makes the implementation of variations and generalisations transparent, as special cards are added to the deck.

This study also outlines a method for exploring the dynamics of a game, and measuring its degree of balance, using a simple evolutionary algorithm. This article specifies tools that have proven useful in academic research in hopes that some or all will also prove useful for game design.

2 Deck-Based PDs

When a mathematical game is transformed into a card game by placing its moves on a limited set of cards, the new game will be called the *deck-based* version of the mathematical game. Thus a *deck-based Prisoner's Dilemma* (DBPD) would use a deck with *C*s and *D*s printed on the cards, and [2] explores a first attempt in this direction. If each player is given only two cards (*C* and *D*) and the game only lasts a single round then we have simply recreated the original mathematical game.

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