From Mathematical Insight to Strategy

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This short note describes how the design process of a group-making game exploited a mathematical insight, based on the constant e, to allow a more intuitive strategy for players.

1 Introduction

OMEGA is a strategy board game invented in 2010,[1] in which players strive to maximise their points using *multiplicative scoring*. The rules are as follows.

Omega is played by P = 2 to 4 players, on a hexagonal tiling of hexagons, which starts the game empty.

1. Players take turns placing one stone of each colour at any empty cell, i.e. P stones must be played each turn.

2. The game ends when a full round of moves by all players can not be completed (some cells may remain empty). All players, therefore, have the same number of moves.

3. Each player's score is the product of the size of each group of their colour. The player with the highest score wins.

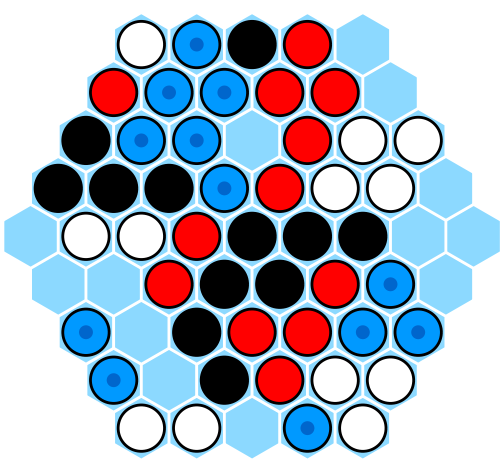

Figure 1. A four-player game of size-5 Omega.

For example, Figure 1 shows a completed four-player game of Omega on a size-5 board, won by White. The final scores are as follows:

White:	$1{\times}2{\times}2{\times}3{\times}4$	=	48
Black:	$1{\times}4{\times}7$	=	28
Red:	$1{\times}2{\times}4{\times}5$	=	40
Blue:	$1{\times}2{\times}3{\times}6$	=	36

1.1 Design Goals

The primary design goal for Omega was to create a game with a balanced winning condition. Instead of players trying to create the most groups or the least groups, or the biggest group or the smallest group, I wanted players to strive for something 'in between' in order to win. This would theoretically allow more subtle strategies. Furthermore, I wanted to make players also play the opponents' stones each turn, so that they did not focus solely on their own formations.

Group creation and multiplicative scoring are not novel mechanisms, but Omega expands on these ideas as follows:

- Fixed number of stones of each colour.
- The number of groups to be multiplied is variable and undefined at the start.

2 Maximising the Score

Given n stones split into two groups, the strategy to maximise their multiplicative score is simply to make both groups equal in size. This is easily demonstrated as follows.

Let a and b be the number of stones of each group, hence $a = n - b$. Maximising the score $a{\cdot}b$ is, therefore, equivalent to maximising $(n-b){\cdot}b = n{\cdot}b - b^2$. Taking the derivative and equating with 0, we have $n - 2{\cdot}b = 0$, which gives $b = \frac{n}{2}$ and $a = \frac{n}{2}$.

Recall, however, that the design goal was to force players to create several balanced groups. The question then becomes: *is there an optimum group size for more than two groups?*

[1] http://www.nestorgames.com/#omega_detail

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