Catch-Up: A Game in Which the Lead Alternates

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Catch-Up is a two-player game in which the players' scores remain close throughout the game, making the eventual winner – if there is one – hard to predict. Because neither player can build up an insurmountable lead, its play creates tension and drama, even between players of different skill. We show how the game is played, demonstrate that its simple rules lead to complex game dynamics, analyse some of its most important properties, and discuss possible extensions.

1 Introduction

I T is a challenge to design interesting two-player games with simple rules that keep the score close, even between players of different skill. When the game score is close, players experience *tension* and *drama* by not knowing too far in advance who will win. This drama has been discussed qualitatively [1, 2] and quantitatively [3].

To enhance tension, games often have catch-up mechanisms, sometimes called *rubber banding* [1]. Players who are behind can receive a boost to help them recover, and players who are ahead are prevented from maintaining or accelerating their lead.

Economists describe the desire to minimise inequality as *inequity aversion*, wherein people prefer rewards to be allocated evenly [4]. Designing games with inequity aversion can create a more balanced competitive experience, allowing experts and novices to enjoy playing together as the score will remain close throughout the game. A game is also often more enjoyable if one is not losing by a large amount. However, too much catching up can lead to games in which the winner is not determined until the very end, making early moves meaningless.

1.1 Catch-Up

We present Catch-Up, a *minimal game* [5] with simple rules that can be learnt quickly, invented by the authors with these ideas in mind.[1] The rules are as follows.

Catch-Up starts with a set of numbers S.

1. Two players, P_1 and P_2, begin with scores, s_1 and s_2, of zero. P_1 starts by removing a number from S, which is added to his or her score.

2. The players then take turns removing one or more numbers from S, one by one, until the acting player's score equals or exceeds the opponent's current score.

3. When this is no longer possible, the acting player receives any remaining numbers. The player with the higher score wins; the game is drawn if scores are tied.

Catch-Up provides meaningful choices, with score balancing built into its rules. Players alternate holding the lead, with the score difference bounded by a relatively small number. Note that the game is actually played with a *multiset* of numbers, i.e. some numbers may be repeated, but we use the term 'set' here for simplicity.
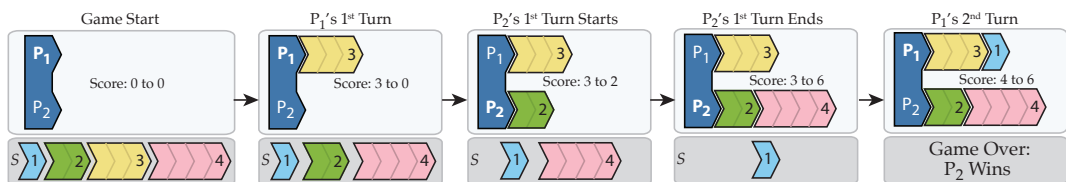

Figure 1. An example game of Catch-Up won by player P_2.