The Double Circle Real 5×5×5

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Twisty puzzles, such as the Rubik's Cube, are typically thought of as geometric solids cut with a set of planes that allow a subset of the puzzle to be rotated. Stickers are applied to their surface such that a solved state is arrived at when each face is restored to a solid colour. This paper considers the 125 individual cubies that make up a 5×5×5 twisty puzzle, and presents a method for including all 125 cubies in the solution, not just the 98 that appear on the surface.

1 Introduction

In 1974, Ernő Rubik invented the 3×3×3 *twisting puzzle*, hereafter referred to as the '3×3×3'. Produced and marketed by Ideal Toys in 1980 as the Rubik's Cube, it launched twisty puzzles into the mainstream. In 1981, Péter Sebestény applied for a patent for the 4×4×4, shown in Figure 1.

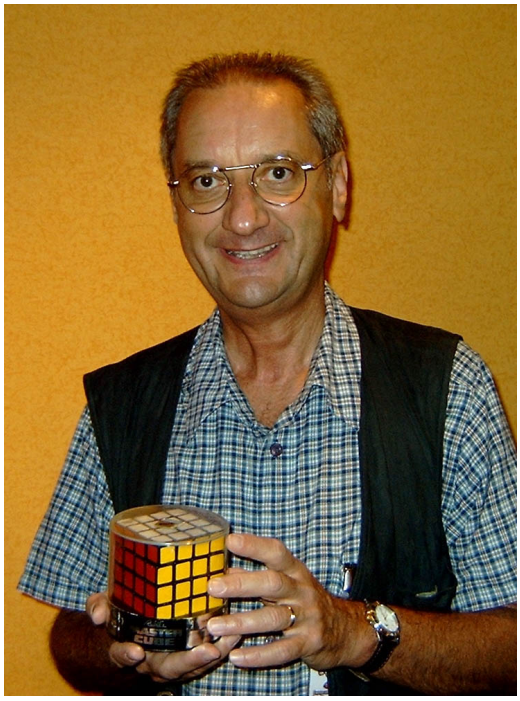

Figure 1. Péter Sebestény with his 4×4×4 cube. Image from speedcubing.com, with permission.

Later that same year, Udo Krell applied for a patent for the 5×5×5, and this trend continues today. Moyu released the 13×13×13 in 2014 and Oskar van Deventer's 17×17×17 [1] has been available from the online 3D printing service Shapeways[1] since 2011.

As we progress to ever higher $N \times N \times N$ cubes, more and more of the implied puzzle is being left hidden and unsolvable. With the 3×3×3, only 26 of the 27 component *cubies* are stickered, i.e. visible to the player. Of these 26, only 20 have a unique position and orientation in the solved state, as the orientation of the face centres is irrelevant to the solution. This problem gets worse for the 4×4×4, which has 8 of its 64 cubies unstickered, and even worse for the 5×5×5, which has 27 of its 125 cubies unstickered.

1.1 Rubik-Type Cube Design

To understand why these internal cubies were left out, we need to examine the design rules in use at the time. Péter Sebestény [2] listed the following design rules for a *Rubik-Type Cube* (RTC):[2]

- The RTC is a regular three-dimensional cube, with edges of equal length.
- The RTC appears to be made of smaller regular unit cubes, the same number of unit cubes along the edges.
- The edges of the unit cubes on the surface of the RTC appear to be of the same length, irrespectively of their unseen shape in the interior.
- The unit cubes that are of the same distance from a freely chosen face of the RTC form a layer. Each layer that is parallel to this chosen face can rotate independently around the axis that is perpendicular to this face and goes through the centre of the RTC.
- No part of the internal structure of the construction can be seen from outside, no matter how the layers are moved.
- The RTC consists of only mechanical components; there are no magnetic or electric parts. It may contain springs.
- The RTC is time-independent and retains its form. It needs no battery that can lose its charge, and can be used at any time. Ageing, wearing off, melting from heat and breakage due to forceful use do not contradict this requirement.

[1] http://www.shapeways.com

[2] Expanded version: https://drive.google.com/file/d/0B8WmsMACU4YubGxuOTJ2RkQ4bkE